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Analysis of Rolling Theory by Analog Computer

— Karman's Differential Equation —

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Analysis of Rolling Theory by Analog Computer
(Karman's Differential Equation)

§1. Introduction

Since Karman's differential equation cannot be solved strictly analytically, Tselikov, Trinks, Nadai, Hill and others obtained its solution through approximation or construction by various means, which, however, require considerable labor. The analysis by analog computer presented here is an automatically converging calculation system using an automatic programming system, markedly saving calculation time, e.g. determination of a pressure distribution curve, roll radius or projected contact length etc. takes only 1 ~ 2 minutes. Besides, in the logical operation and control of analog computer, reverse time operation system is adopted, widely differing from the conventional boundary value problem. The outline of this analysis will be described below.

§2. Outlines of Rolling

Assume that a plate of thickness h_1 is rolled to thickness h_2. While circumferential speed of roll, v, is constant everywhere, material speed at roll exit v'_2 is always faster than that at roll entrance v'_1, since the material is elongated by rolling. In Fig. 1, therefore, material speed of hatched area A is slower than v, and material is drawn in toward roll exit by frictional force against the roll face, while material speed of hatched area B is faster than v, preventing flow of material due to frictional force against the roll face. At the mid-point between A and B, e.g., Y in Fig. 1, material speed is equal to v, eliminating mutual slipping. Hence point Y is called neutral point or watershed, and ϕ, roll angle at neutral point.

The absolute value of frictional force between roll and material is represented by a single-peaked curve with maximum at neutral point, as illustrated in Fig. 1.

When horizontal pressure q is applied to a block of material which is deformed by vertical pressure p as shown in Fig. 2, deformation requires vertical pressure p+q to overcome the effect of horizontal pressure q. Accordingly, distribution of vertical pressure takes thatch-shaped curve as shown in Fig. 1.

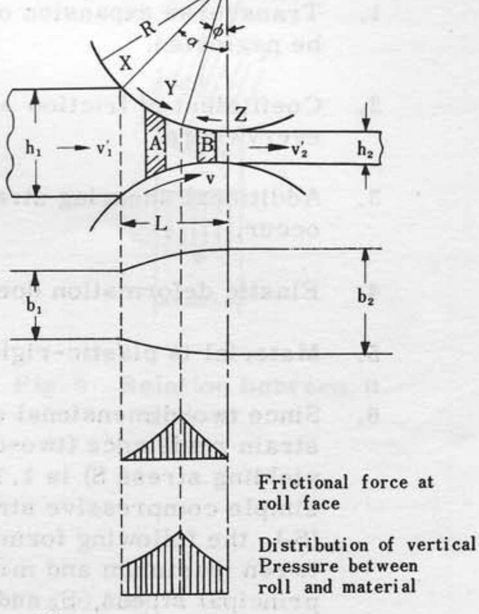


Fig. 1. Rolling Process

§3. Derivation of Equation

3-1. Projected Contact Length

If the roll is assumed not to deform in the course of rolling, projected contact length L (to be represented by X hereinafter) is given by Eq. (1), exactly, or by Eq. (2), in approximation.

$$X_1 = \sqrt{R\triangle h - \frac{(\triangle h)^2}{4}} \qquad (1)$$

$$X_1 = \sqrt{R\triangle h} \qquad (2)$$

In case that the segment of roll in contact with material is elastically deformed in the course of rolling, such as cold rolling of thin plate, the actual projected contact length is longer than X_1, as given by Hitchcock's formula, Eq. (3)

$$X_1 = \sqrt{\left[\frac{8R(1-\nu^2)p'm}{\pi E}\right]^2 + R\triangle h} + \frac{8R(1-\nu^2)p'm}{\pi E} \qquad (3)$$

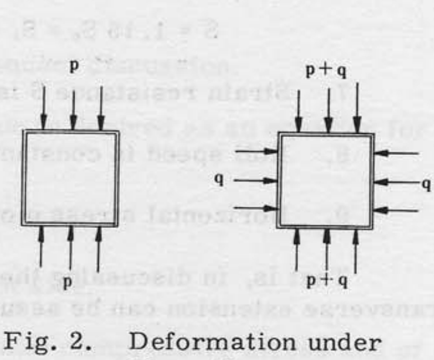


Fig. 2. Deformation under Plane Stress

Or, more frequently, using modified roll radius R' caused by elastic deformation, which can be represented by Eq. (4) according to Hitachicock, X ' may be represented by Eq. (5) in place of Eq. (3).

$$\frac{R'}{R} = 1 + \frac{16(1-\gamma^2)}{\pi E}\,\frac{p}{b\triangle h} \qquad (4)$$

$$X' = \sqrt{R'\triangle h} \qquad (5)$$

where γ: Poisson's ratio
E: Young's modulus
b: Width of material
pm: mean rolling pressure
P: rolling load

Rolling load P is represented by Eq. (6) as a function of rolling pressure p as in Eq. (6).

$$P = b\int_0^{X_1'} p\,dx \qquad (6)$$

3-2. Karman's Differential Equation

In discussion of stress equilibrium at a minute segment bounded by sections parallel to roll shaft, as the hatched area in Fig. 3, Karman derived a differential equation in the following way, assuming 9 hypotheses given below.

1. Transverse expansion of material can be neglected.

2. Coefficient of friction μ is uniform everywhere.

3. Additional shearing strain does not occur.

4. Elastic deformation does not occur.

5. Material is plastic-rigid.

6. Since two-dimensional constraint strain resistance (two-dimensional yielding stress S) is 1.15 times simple compressive strain resistance (S_0), the following formula holds between maximum and minimum principal stress, S_1 and S_3, respectively.

$$S = 1.15\,S_0 = S_1 - S_3$$

7. Strain resistance S is uniform throughout arc of contact.

8. Roll speed is constant.

9. Horizontal stress q of material distributes uniformly in the direction of thickness.

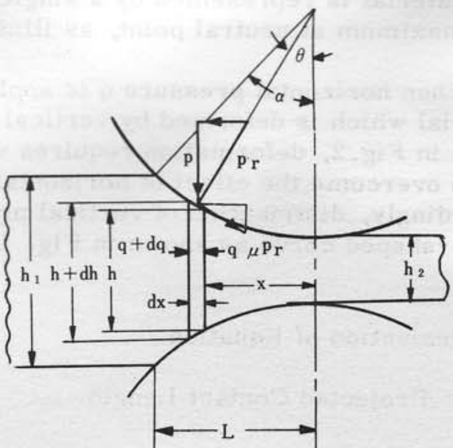


Fig. 3 Pressure Equilibrium in Rolling

That is, in discussing the equilibrium of horizontal stress at the hatched area in Fig. 3, transverse extension can be assumed always 1, to omit calculation.

Horizontal stress $\triangle Q$ acting on the small portion of material through the cross-section is:

$$\triangle Q = (h+dh)(q+dq) - h \cdot q$$

Hence, $\triangle Q = hq + dh \cdot q + dq \cdot h + dh \cdot dq - hq$

$$\doteqdot dh \cdot q + dq \cdot h$$

$$= d(h \cdot q) \qquad (7)$$

Since horizontal component $\triangle Q$ of stress acting on the small portion of material through the surface is the sum of horizontal component of pressure vertical to roll face pr and of horizontal component of frictional force pr acting to the surface of material, horizontal stress on the one side of material $\triangle Q/2$ is given by (8), (8') or (8'').

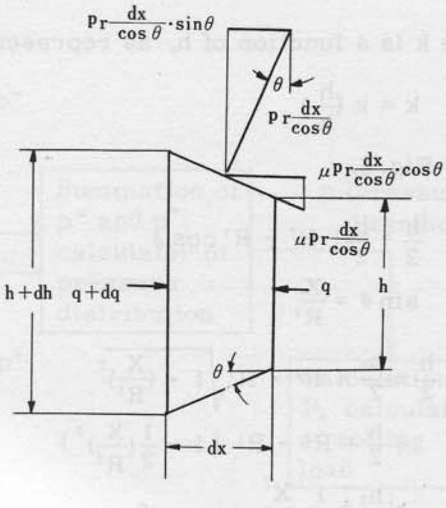


Fig. 4 Details of Stress Equilibrium

$$\frac{\triangle Q}{2} = \left(\rho_r \frac{dx}{\cos\theta}\right) \sin\theta - \mu\left(\rho_r \frac{dx}{\cos\theta}\right) \cos\theta \qquad (8)$$

$$= \rho_r (\tan\theta - \tan f)\, dx \qquad (8')$$

where $\mu = \tan\theta$.

While Fig. 4 gives stress equilibrium at the entrance of roll, the direction of frictional force is reversed at the exit of roll, thus giving Eq. (8'') from Eq. (8').

$$\frac{\Delta Q}{2} = \rho_r (\tan \theta \mp \tan f)\, dx \qquad (8'')$$

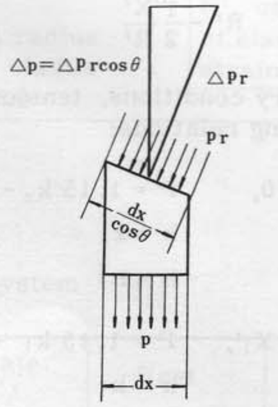


Fig. 5 Relation between p and pr

Furthermore, putting compressive force and compressive stress acting on material in dx from the roll surface in Fig. 5 as $\triangle$pr and pr, respectively, the following relation holds:

$$\triangle pr = pr \frac{dx}{\cos\theta} \qquad (9)$$

If vertical component of $\triangle$pr is put as $\triangle p = \triangle pr \cos\theta$ and compressive stress in the vertical direction as p

$$\triangle p = \triangle pr \cos\theta = p \cdot dx \qquad (10)$$

Accordingly, the following relation always holds from Eq. (9) and (10),

$$p = pr \qquad (11)$$

Hence, p and pr will not be distinguished in the subsequent discussion.

From Eqs. (7) and (8'), Karman's differential equation is derived as an equation for equilibrium of $\triangle Q$ as below:

$$d\left(\frac{h \cdot q}{2}\right) = p(\tan\theta \mp \tan f)\, dx \qquad (12)$$

where minus sign refers to entrance side and plus sign to exit side.

Since vertical compressive stress p is sum of horizontal compressive stress and of stress due to two-dimensional constraint strain resistance,

$$q + k = p \tag{13}$$

where k is a function of h, as represented below

$$k = k\left(\frac{h}{h_1}\right) \tag{14}$$

From Fig. 6,

$$\frac{h}{2} = \frac{h_2}{2} + R' - R'\cos\theta$$

$$\sin\theta = \frac{X}{R'}$$

$$\text{hence } \frac{h}{2} = \frac{h_2}{2} + R' - R'\sqrt{1 - \left(\frac{X}{R'}\right)^2}$$

$$\doteqdot \frac{h}{2} + R' - R'\left\{1 - \frac{1}{2}\left(\frac{X}{R'}\right)^2\right\}$$

$$= \frac{h_2}{2} + \frac{1}{2}\frac{X^2}{R'} \tag{15}$$

$$\text{And } \tan\theta = \frac{X}{R'\cos\theta}$$

$$= \frac{X}{R' - \frac{1}{2}\frac{X^2}{R'}} \tag{16}$$

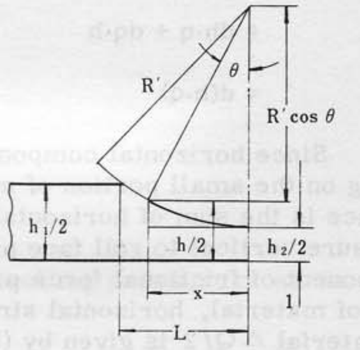


Fig. 6 Deformation Process of Material

As boundary conditions, tension to material at entrance and exit, σ_1 and σ_2, should satisfy the following relations:

where $X = 0$, $P = 1.15\,k_2 - \sigma_2 = P_2$, $k = k_2$, $h = h_2$ (17)

where $X = X_1'$, $P = 1.15\,k_1 - \sigma_1 = P_1$, $k = k_1$, $h = h_1$ (18)

§ 4. Operation of Analog Computer

4-1. Construction of Automatic Program

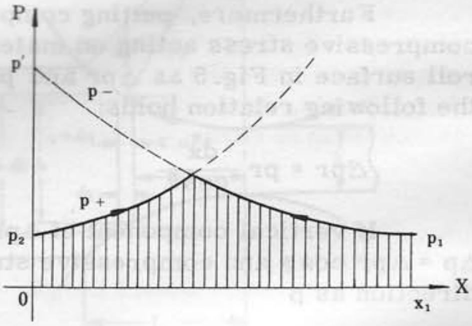


Fig. 7. Pressure distribution

Since pressure distribution p is as illustrated in Fig. 7, for projected contact length X_1 of roll without elastic strain, p^- value is calculated starting from the given initial value p_1 in the direction of $X_1 \rightarrow 0$, and the value p' at $X = 0$ is memorized.

Then at $X = 0$, giving initial values p_2 for p^+ and p' for p^-, calculating simultaneously in the direction $0 \rightarrow X_1$, to obtain pressure distribution curve p as shown by thick lines in Fig. 7. At the same time, determining rolling load P, radius R' of roll with elastic strain is obtained from Eq. (4). From R' and Eq. (5), new projected contact length X' is determined, and with this the above calculation is repeated in the same way. By repeating these trial calculations, X', R', p and P converge to finite values, which are solutions of Karman's differential equation.

A flow chart for the automatic program and time chart for its control process are given below.

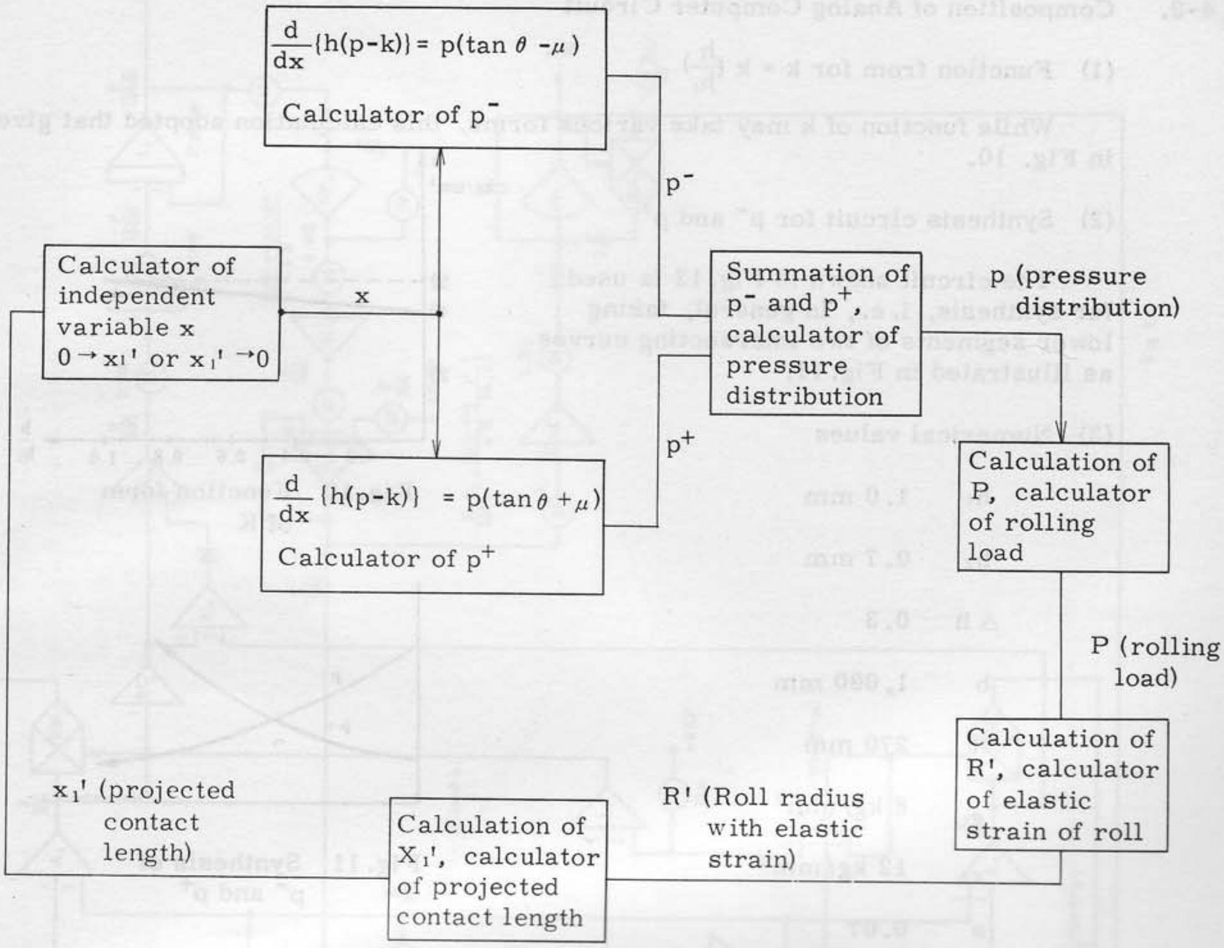


Fig. 8 Automatic Program System

X - calculator	Calculate $X_1' \to 0$	HOLD at X = 0	Calculate $0 \to X_1'$	Rest	Rest
p^--calculator	Calculate in reversed time from X = X'	HOLD at X = 0	Calculate in normal time from X = 0	Rest	Rest
p^+-calculator	Rest	Rest	Calculate in normal time from X = 0	Rest	Rest
p-calculator	Rest	Rest	Calculate	Rest	Rest
P-calculator	Rest	Rest	Calculate	HOLD	Rest
R'-calculator	Rest	Rest	Calculate	HOLD	Rest
X_1'-calculator	HOLD	HOLD	HOLD	Calculate	HOLD

Fig. 9 Control Process at the Program Controller

4-2. Composition of Analog Computer Circuit

(1) Function from for $k = k\left(\frac{h}{h_1}\right)$

While function of k may take various forms, this calculation adopted that given in Fig. 10.

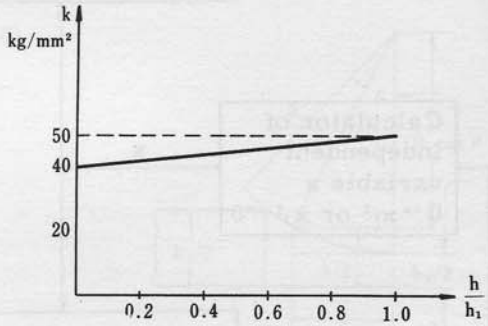


Fig. 10 Function form of K

(2) Synthesis circuit for p^- and p^+

The circuit shown in Fig. 12 is used for synthesis, i.e., in general, taking lower segments of two intersecting curves as illustrated in Fig. 11.

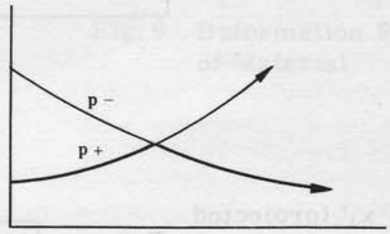


Fig. 11 Synthesis of p^- and p^+

(3) Numerical values

h_1	1.0 mm
h_2	0.7 mm
$\triangle h$	0.3
b	1,000 mm
R	270 mm
σ_1	8 kg/mm^2
σ_2	12 kg/mm^2
μ	0.07
E	2.1×10^4 kg/mm^2
ν	0.3

(4) Block diagram of analog computer

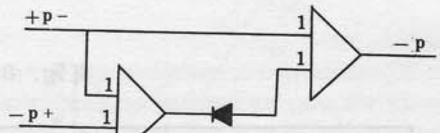
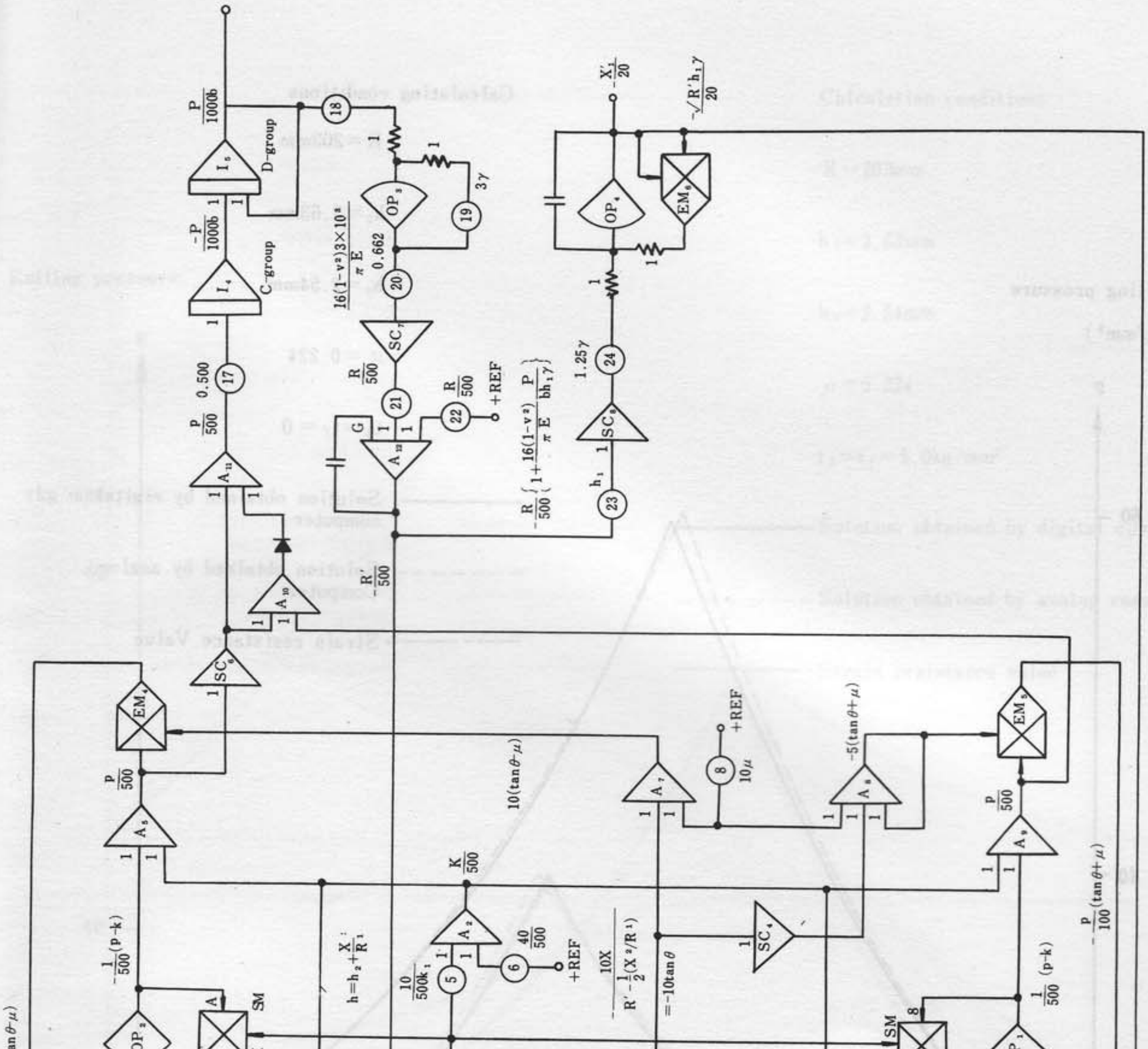


Fig. 12 Circuit for Synthesis

Block diagram of the analog computer is given in Fig. 13, in which there are five integrators controlled separately by the sequence shown in Fig. 14.

Control time	3 ~ 5 sec.	X_1' sec/mm	3 ~ 5 sec	X_1' sec/mm	3 ~ 5 sec
Integrator in A-group	RESET	COMPUTE	HOLD	COMPUTE	RESET
Integrator in B-group	RESET	RESET	RESET	COMPUTE	RESET
Integrator in C-group	RESET	RESET	RESET	COMPUTE	HOLD
Integrator in D-group	HOLD	HOLD	HOLD	HOLD	COMPUTE
Action of relay K_1	OFF	OFF	ON	ON	OFF

Fig. 14 Operation control sequence for program controller



Program Contrller
No.4 Shift Term
control signal
Program Controller
No.2 Shift Term
control signal
A-qroup
B-group
C-group
D-group
+REF
-REF
+PEF
ON
OFF

Fig 13

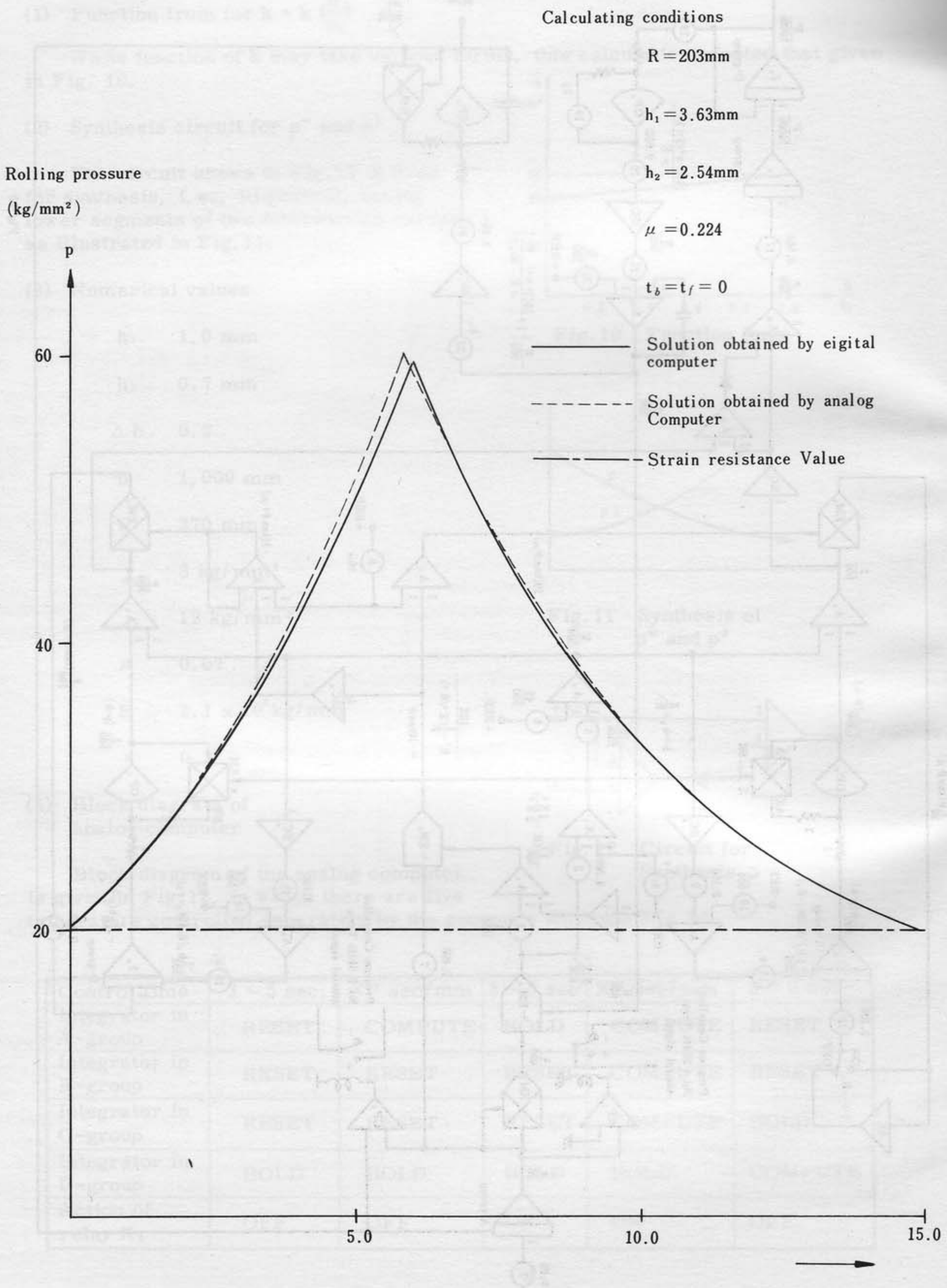


Fig 15 Exampls of digital and analog computation ①

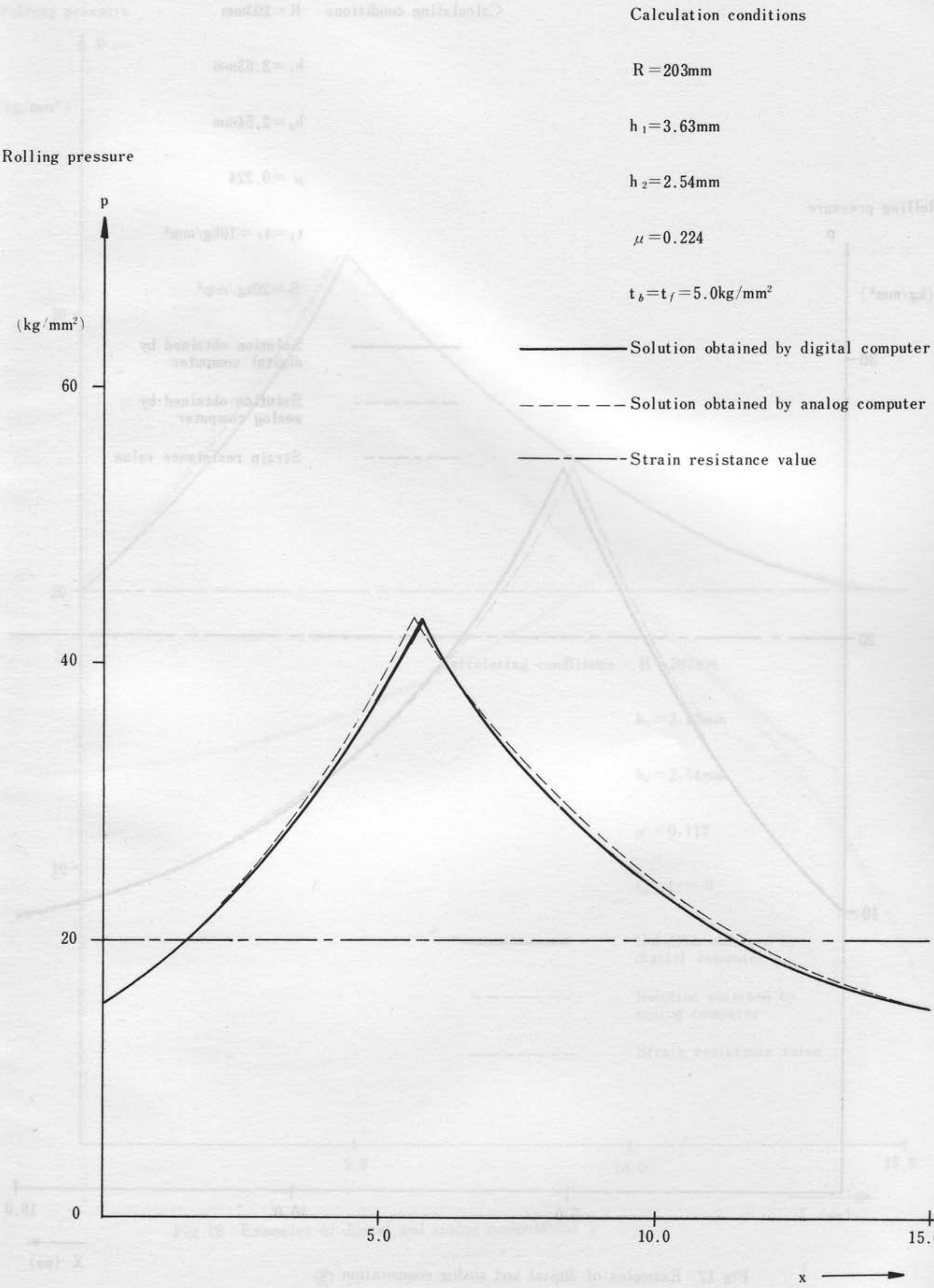


Fig 16 Examples of digital and analog computation ②

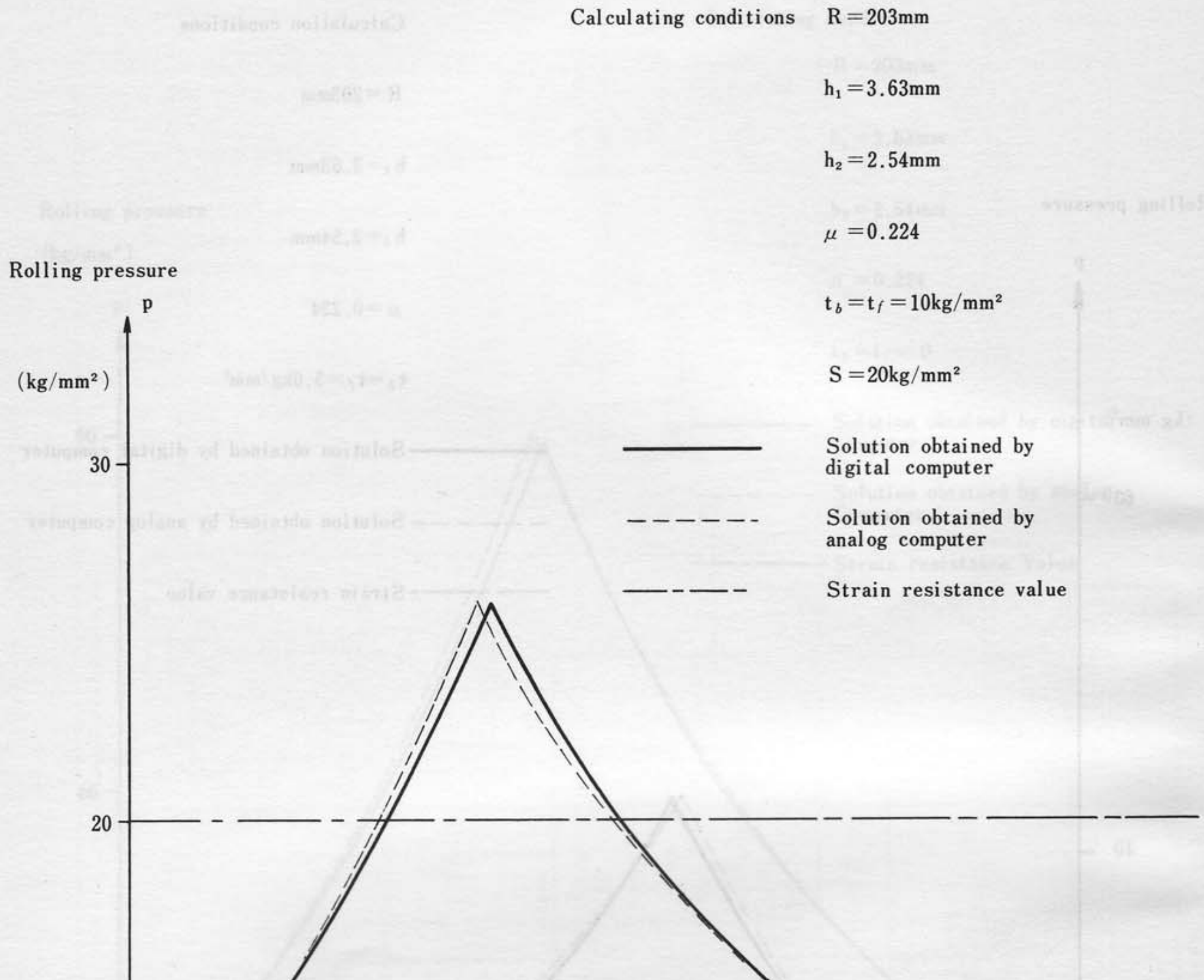


Fig 17 Examples of digital and analog computation ③

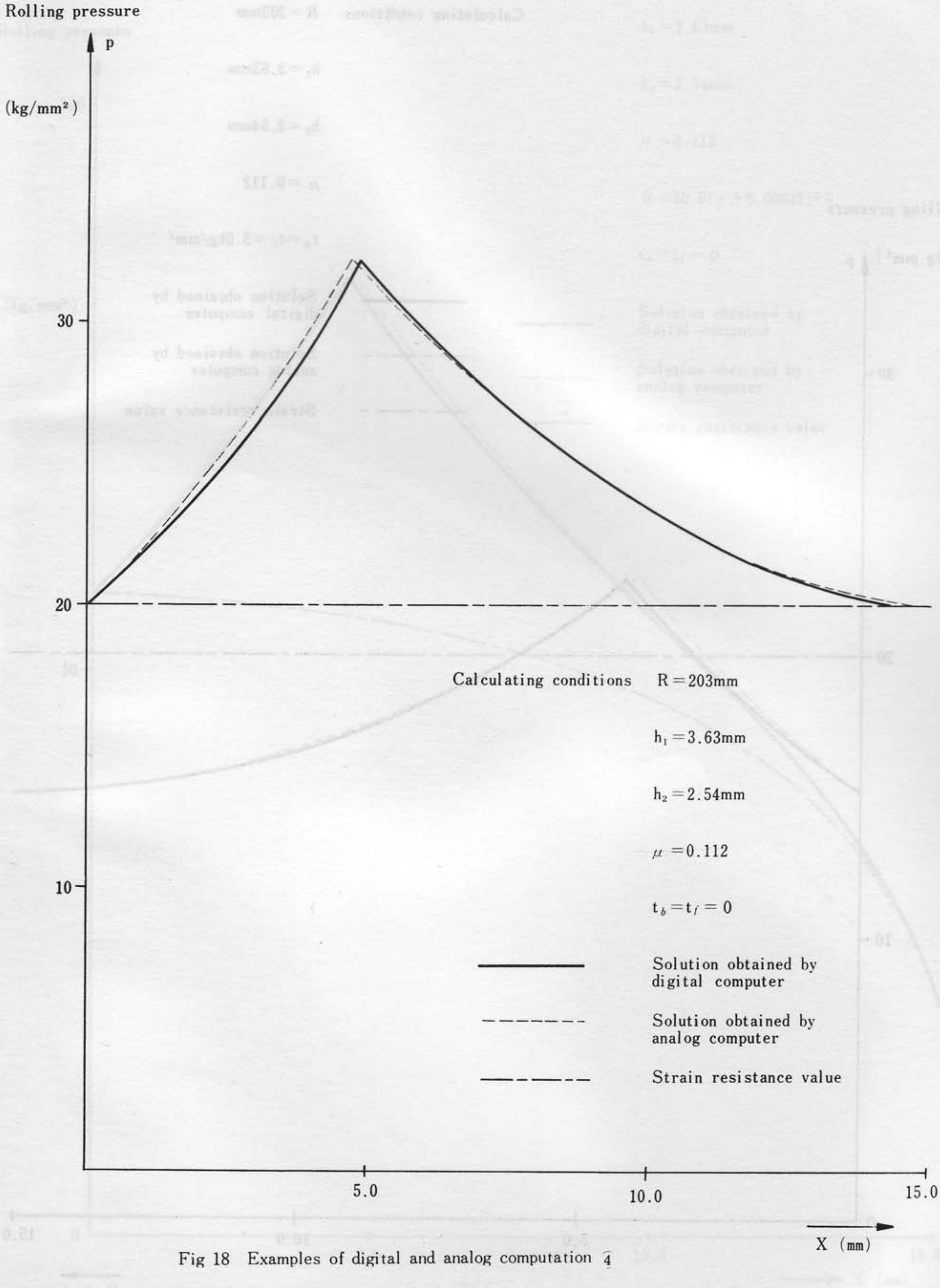


Fig 18 Examples of digital and analog computation 4

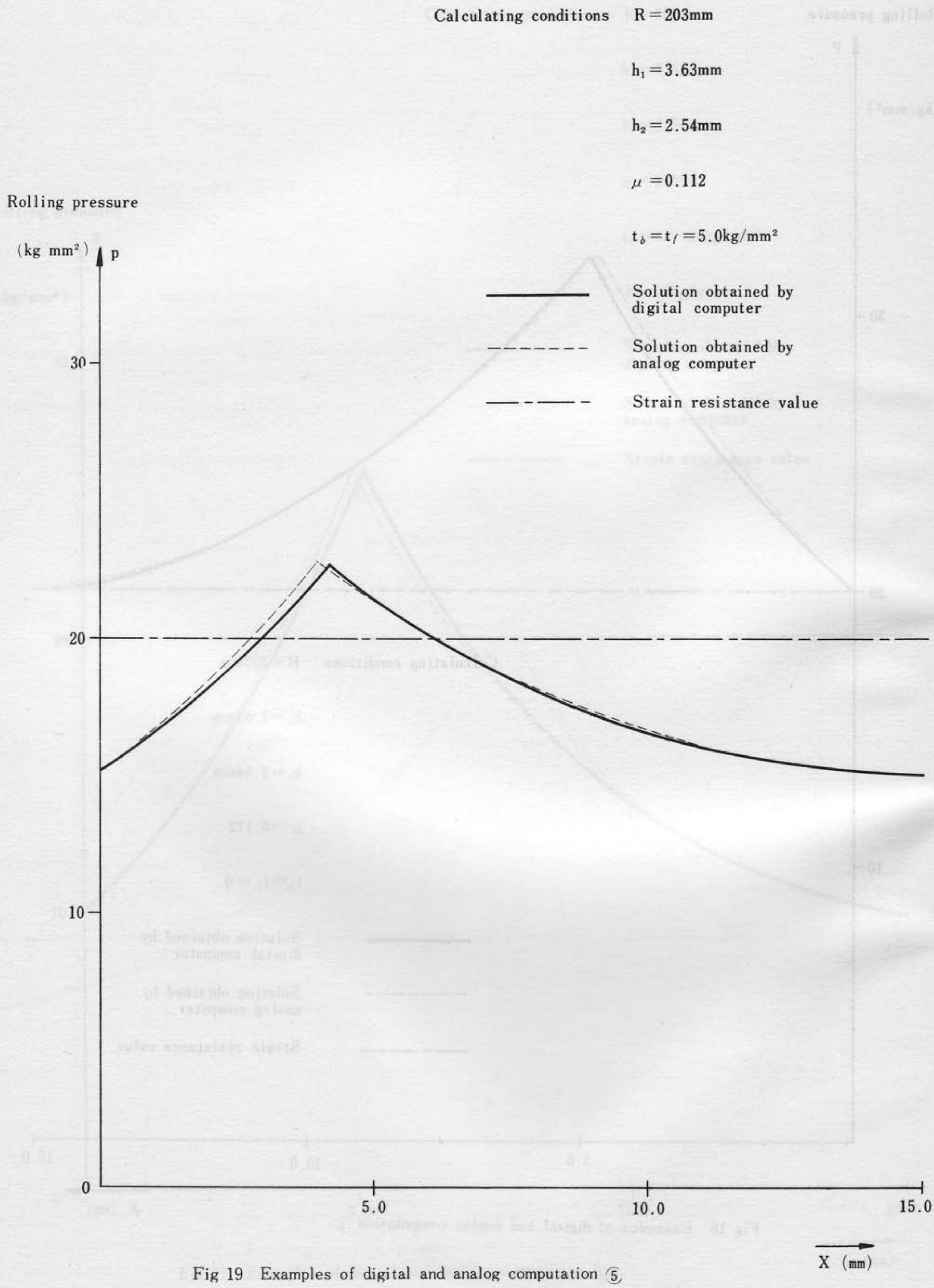


Fig 19 Examples of digital and analog computation ⑤

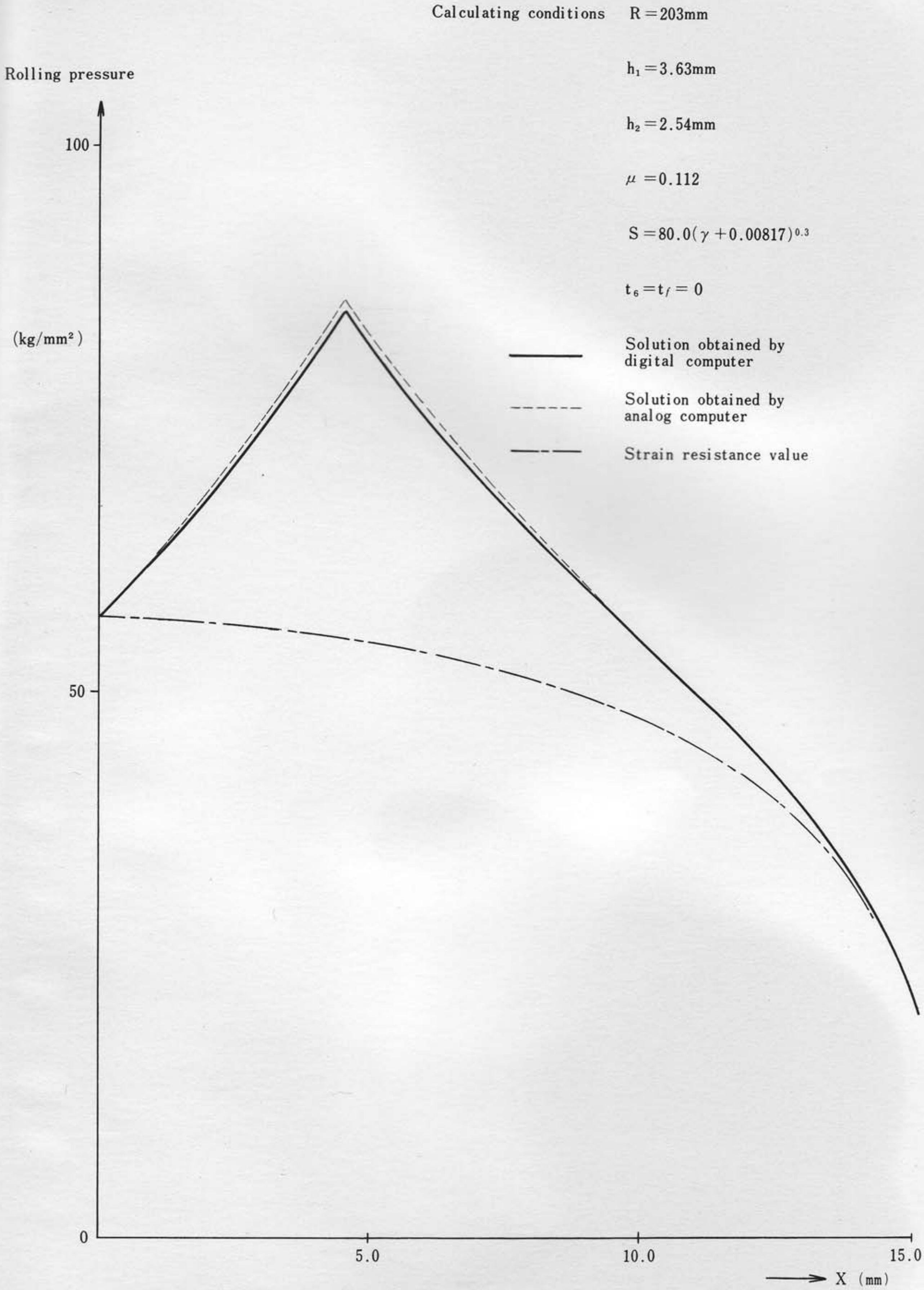


Fig 20 Examples of digital and analog computation 6